# Dust-Acoustic Waves Driven by an Ion-Dust Streaming Instability in Laboratory Discharge Dusty Plasma Experiments

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## ABSTRACT

Dust acoustic waves (DAW) are spontaneously excited in dusty plasmas produced in dc and rf discharge plasmas over a wide range of plasma and dust conditions. A common feature of these plasmas is the presence of an ion drift relative to the dust, which is driven by an electric field, $E_o$ in the discharge. Using a three fluid model of the DAWs, including the zero order electric field and collisions of all species with the background neutral gas (pressure $P_o$), DAW stability curves were obtained in the $E_o$–$P_o$ plane, for various dust and wave parameters. The ($E_o$, $P_o$) data points from several experiments in which DAWs have been observed are also shown in comparison to the theoretical stability boundaries. This analysis supports the conclusion that the DAWs are excited by an ion-dust streaming instability.

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The dust acoustic wave (DAW) is a long wavelength, low frequency collective oscillation mode in a dusty plasma in which the massive, charged particles participate in the wave motion. The first linear *and* nonlinear analysis of this mode was presented by Rao, Shukla and Yu, who obtained the linear dispersion relation.[1] The conditions for excitation of the DAW in a collisionless dusty plasma were first obtained, using a standard Vlasov approach, by Rosenberg[2] who showed that the DAW could be driven unstable by ion and electron drifts greater than the DA phase speed, which is typically much less than the ion thermal speed. Since laboratory dusty plasmas are generally weakly ionized, the effects of collisions with the neutral atoms were subsequently included in the Vlasov analysis.[3] A fluid treatment of the current-driven DA instability in a collisional dusty plasma was carried out by D'Angelo and Merlino[4] in which the DAWs were driven unstable by electrons and ions drifting in a zero order, dc electric field. Joyce, *et al*.,[5] showed that an ion-dust two stream instability, which occurs below a critical neutral pressure, can heat the dust particles and prevent condensation into dust crystals.

DAWs have been observed in several experiments, (for example, see refs. 6-17), and under a variety of plasma source and dust conditions. Two main plasma sources have been used – the dc glow discharge[7, 9-11, 13, 17] and the capacitively coupled rf discharge.[6,12, 15, 16] However, hot filament discharges[8], inductively coupled rf discharges[12] and Q machine plasmas[7] have also been used. One experiment (ref. 14) was conducted under microgravity conditions. A magnetic field was used in the plasmas of references 7, 8, and 15. In one experiment (ref. 6), the dust particles were grown in the discharge using a reactive gas mixture, all other experiments used dust particles that were dispersed into the plasma. These experiments have in common the presence of spontaneously excited DAWs, weakly ionized plasmas, and drifting ions. Table I contains a summary of the various parameters pertinent to the experiments.

This Brief Communication attempts to understand the ubiquitous nature of DAWs in laboratory dusty plasmas. The relevant data from experimental observations of unstable DAWs will be compared to stability curves obtained from a simple one-dimensional fluid model of current-driven DAWs in a collisional dusty plasma,[4,18] which will be briefly summarized. The electrons, ions and dust are treated as fluids obeying the continuity and momentum equations: $\partial n_j/\partial t + \partial(n_j u_j)/\partial x = 0$,

$m_j n_j\left(\partial u_j/\partial t + u_j\,\partial u_j/\partial x\right) - kT_j\left(\partial n_j/\partial x\right) - q_j n_j E = -m_j n_j \nu_{jn} u_j$, where, $j = (e, i, d)$ refers to ions, electrons and dust, respectively, $q_j = (e, -e, -eZ)$, where $Z$ is the dust charge number (assumed to be fixed), and $\nu_{jn}$ is the collision frequency of species $j$ with the background neutrals. For the electrons and ions, the collision frequencies are given by: $\nu_{en} = N\sigma_{en}V_{eT}$ and $\nu_{in} = N\sigma_{in}V_{iT}$, where, $\sigma_{en}$ and $\sigma_{in}$ are the electron-neutral and ion-neutral collision cross sections, N is the neutral density, and $V_{eT}$ and $V_{iT}$ are the electron and ion thermal speeds. The dust-neutral collision frequency is given by the Epstein formula, $\nu_{dn} = 4\pi a^2 N V_{nT}\, m_n/3m_d$, where $a$ is the dust grain radius, $m_d$ is the mass of the dust grains, and $m_n$ and $V_{nT}$ are the mass and temperature of the neutrals. A uniform and time independent electric field, $E_o$, which gives rise to zero-order fluid drifts, $u_{jo} = q_jE_o/(m_j\,\nu_{jn})$, is included in the equilibrium, and the zero-order particle densities are related by the charge neutrality condition, $n_{io} = n_{eo} + Zn_{do}$. The dispersion relation relating the complex wave angular frequency (ω) to the wavenumber (K) was obtained by linearizing the fluid equations, using Gauss's law, $\partial E/\partial x = \left(e/\varepsilon_o\right)\left(n_i - n_e - Zn_d\right)$ to relate the wave electric field to the first order charge densities, and assuming all first-order quantities vary as $\exp[i(Kx-\omega t)]$: $1-\sum_{j=e,i,d}\omega_{pj}^2/A_j = 0$, where, $\omega_{pj} = \left(n_j q_j^2/\varepsilon_o m_j\right)^{1/2}$, and

$A_j = (\omega - Ku_{jo})(\omega - Ku_{jo} + i\nu_{jn}) - K^2V_{jT}^2$. For a given set of parameters, the dispersion relation was solved numerically for the stability curve in the $E_o - P_o$ plane. The following fixed parameter values were used: $m_i = m_n = A(1.67\times10^{-27}$ kg), $kT_e = 2.5$ eV, $kT_i = kT_n = 0.03$ eV, $\sigma_{en} = \sigma_{in}/10 = 5\times10^{-20}$ m$^2$, $n_{i0} = 5000\ n_{d0} = 5\times10^{14}$ m$^{-3}$, N(m$^{-3}$) $= 3.3\times10^{19}$ $P_o$(mtorr). Stability plots ($E_o$ vs. $P_o$) were computed for various values of the dust radius (which for a fixed $T_e$, determines the dust charge), dust temperature, $T_d$, and the wavelength of the DAWs. The chosen parameters represent a reasonable range that includes most of the conditions in the experiments that are considered. The intention here is not to make a detailed comparison between the experiments and theory but to determine where on the stability plot (stable or unstable) the experimental ($E_o$, $P_o$) points lie. The stability curves are shown in Fig. 1 for four cases:

(1) $A = 20$, $\lambda = 5$ mm, $a = 1$ μm ($Z = 4000$), $T_d = 0.03$ eV;

(2) $A = 40$, $\lambda = 5$ mm, $a = 5$ μm ($Z = 2\times10^4$), $T_d = 0.03$ eV;

(3) $A = 40$, $\lambda = 10$ mm, $a = 0.5$ μm ($Z = 2000$), $T_d = 30.0$ eV

(4) $A = 40$, $\lambda = 1$ mm, $a = 0.5$ μm ($Z = 2000$), $T_d = 0.03$ eV.

Each curve represents a stability boundary, with points above (below) curve representing ($E_o$, $P_o$) values for which the DAW is *stable* (*unstable*) for that particular set of parameters. The ($E_o$, $P_o$) values corresponding to the experiments listed in Table I are also shown. All the experimental data points fall *below* the curves indicating that, according to the theory, the DAWs should indeed be unstable. Note that the data for ref. 10 (using neon gas)) does lie below curve (2) which corresponds to a neon plasma. One additional point to mention is that in refs. 6, 10, 12, and 13, it was pointed out that the DAWs only appeared when the pressure was reduced below some critical value, or that DA waves could be quenched by raising the neutral pressure. Also, further measurements following on the work in ref. 9, showed that the DAWs were also quenched if the discharge current was decreased below a critical value ~ 1 mA.[19]

When comparing the theoretical results with the experimental data one should keep in mind that the experimental values for $E_o$ should be considered as estimates only. The electric field cannot be directly measured in a dusty plasma using a Langmuir or emissive probe because of the disturbance that probes cause on the dust cloud. In some cases, electric potential measurements were made without the dust present and the obtained values of $E_o$ were taken to be representative of those with the dust present. In the absence of any direct measurement, a value for $E_o$ can be estimated using the fact that, in some cases, the electric field which drives the ion drift also provides the levitation force for the dust, so that $E_o$ can be estimated from the equilibrium condition $eZE_o = m_d g$. In addition, we have assumed a spatially uniform electric field which is probably not the case in the experiments.

It is instructive to compare the ion drift speeds in the experiments considered with typical values of the dust acoustic speed, $C_{da}$ and the ion thermal speed $V_{iT} \sim (2kT_i/m_i)^{1/2}$.

The ion drift speed, $u_{io}$ can be estimated from $u_{io} = \mu_i E_o$, where $\mu_i$ is the ion mobility,[20] so that $u_{io}$ is a function of $E_o/P_o$. For the experiments in Table I, $u_{io}/V_{iT}$ is in the range 0.1 −10, significantly above $C_{da}$ which is typically ~ (0.01−0.2) m/s. The critical drift speeds in the experiments exceed $C_{da}$ by 1 to 3 orders of magnitude. It is noted that, in the absence of collisions, instability occurs for[2] $u_{io} > \omega/K \sim C_{da}$.

The possible role of ion streaming in excitation of the DA instability can be examined by comparing the energy density of the streaming ions, $W_{io} = m_i n_{io} u_{io}^2 / 2$ with the DAW energy density, $W_{DA} = W_{DA,p} + W_{DA,f}$, where $W_{DA,p}$ and $W_{DA,f}$ are the energy densities in the particle motion and electrostatic wave fields. For long wavelength acoustic modes, the energy in the field is much smaller than the energy in the coherent particle motion, so that $W_{DA} \approx W_{DA,p} \approx m_d n_{do} u_{d1}^2 / 2$, where $u_{d1}$ is the perturbed dust velocity in the wave which can be related to the perturbed dust density, $n_{d1}$ using the linearized continuity equation, as $u_{d1} = (\omega / K)(n_{d1} / n_{do})$. Since $\omega / K \sim C_{da} = \lambda_{Di}\omega_{pd}$, where $\lambda_{Di}$ is the ion Debye length and $\omega_{pd}$ is the dust plasma frequency, the energy density in the DAW can be written as $W_{DA} \approx m_d n_{do} \lambda_{Di}^2 \omega_{pd}^2 (n_{d1} / n_{do})^2 / 2$, where $n_{d1}/n_{do}$ is the wave amplitude. Using the expressions for $W_{io}$ and $W_{DA}$ we obtain, the ratio $W_{io} / W_{DA} \approx \left[ (n_{io} / n_{do})^2 (u_{io} / V_{iT})^2 \right] \Big/ \left[ Z^2 (n_{d1} / n_{do})^2 \right]$. A numerical estimate of this ratio can be made using the following typical values: $u_{io}/V_{iT} \sim 1$, $n_{io}/n_{do} \sim 10^4$, $Z \sim 5000$, $n_{d1}/n_{d0} \sim 0.1$ (corresponding to the linear growth phase); then $W_{io}/W_{DA} \sim 400$. This indicates that at least for these typical parameter values, there is sufficient free energy in the streaming ions to drive the DAW instability. Although the electrons are also drifting relative to the dust, the energy density in the drifting electron component is down by a factor of $n_{eo}/n_{io}$ as compared to the ions, since $n_{eo}/n_{io} \ll 1$ in a dusty plasma due to depletion of the electrons on the dust.

The results of the stability analysis presented in Fig. 1, and the global energy considerations together with the experimental fact that the DAWs are observed to propagate in the direction of the ion drift (or at angles < 90° to the ion drift[14]), support the conclusion that DAWs observed in discharge plasmas are excited by an ion-dust streaming instability.

Simple considerations of the basic physics of weakly ionized electrical discharges may help provide an insight into why the DA wave is so ubiquitous in these plasmas. The discharge current $I_{dis}$ and the electric field $E_o$ are related by Ohm's law, $I_{dis} = \sigma E_o A$, where $\sigma$ is the plasma conductivity, and A is the cross-sectional area of the discharge. With $\sigma$ taken from Raizer[21] and with $n_e = 5\times10^{14}$ $m^{-3}$, P = 0.2 Torr of argon, a discharge radius of 2 cm, and discharge current $I_{dis}$ ~ (1 – 10 ) mA, we find $E_o$ ~ (60 – 600 ) V/m or E/P ~ 0.3 – 3, which is within the range at which DAWs are typically observed.

In summary, stability curves in the $E_o$ – $P_o$ plane have been obtained from a fluid model for DAWs and compared to observations from a number of experiments. Although some of the experimental reports also included comparisons to various DAW models, it is instructive to compare the results from a diverse set of experiments to one simple model. For the available cases, the experimental ($E_o$, $P_o$) values lie in the region of predicted instability indicating the likely role of ion – dust streaming in driving the DAWs. Further estimates indicate that the ion streaming is a sufficient free energy source for wave excitation. Finally, the characteristics of the electrical discharges used to produce dusty plasmas would seem to make them ideally suited to DA wave excitation. Considering the results in Fig.1, it would appear that pressures ~ 1 Torr (133 Pa) or greater would be required to suppress DAW excitation in laboratory discharge plasmas. Under these conditions the dust tends to condense into the crystalline phase.[5]

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**FIGURE CAPTION**

Figure. 1. (Color online) The curves are the DAW stability boundaries in the $E_o - P_o$ plane for various cases that are described in the text. Points above (below) a curve represent stable (unstable) DAWs. The data points refer to individual experiments in which unstable DAWs were observed.

TABLE I. Parameters of dust acoustic wave experiments.
Unless noted, the gas was argon.

| ref. | plasma | a [μm] | Z | $E_o$ [V/m] | $P_o$ [mtorr] | λ [mm] |
|---|---|---|---|---|---|---|
| 6. | rf capacitive | 1[a] | 4900 | 100 | 3 | 5 |
| 7. | dc glow[b] | 5[c] | $4\times10^4$ | 400 | 70 | 6 |
| 8. | hot filament | 0.15[d] | 2000 | 400 | 1 | 10 |
| 9. | dc glow[e] | 0.4[f] | 1300 | 1000 | 100 | 6 |
| 10. | dc glow[g] | 0.94 | 2500 | 300 | 1000 | 1 |
| 11. | dc glow[g] | 0.94 | 1800 | 180 | 200 | 1 |
| 12. | rf inductive[g] | 0.94 | 2160 | 400 | 375 | 0.7 |
| 13. | dc glow | 0.25[h] | 3000 | 145 | 75 | 5 |
| 14. | rf capacitive[i] | 3.4 | 4000 | 1060[j] | 113 | 1.6 |
| 15. | dc glow[k] | 0.47 | 3100 | 200 | 19 | 4 |
| 16. | rf capacitive | 0.64 | 2300 | 135 | 173 | 1.7 |
| 17. | dc glow | 0.75 | 3000 | 100 | 72 | 6 |

[a] estimated, since dust particles were grown in the plasma

[b] anode glow discharge in $N_2$ with magnetized potassium Q machine source

[c] average size of aluminum silicate powder used

[d] average size of alumina powder used

[e] weakly magnetized anode glow plasma in $N_2$

[f] average size of aluminum silicate particles collected in the plasma

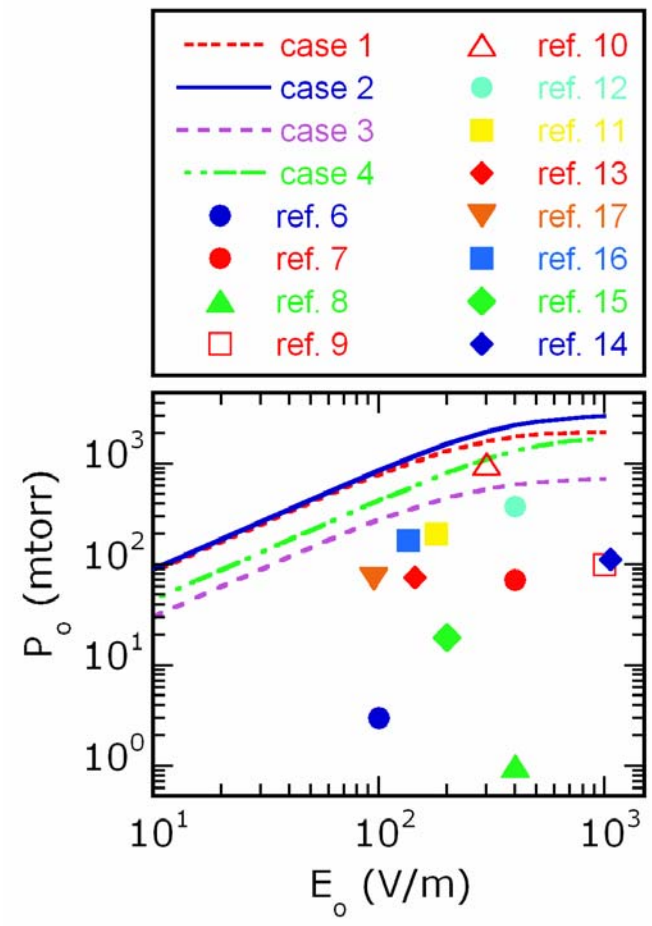
[g] neon plasma

[h] average size of aluminum silicate powder used

[i] experiments conducted microgravity conditions

[j] assumes ions drift $v_o = 0.3\,(kT_e/m_i)^{1/2}$ with $E_o = v_o/\mu_i$, and $\mu_i$ given in ref. 20.

[k] weakly magnetized anodic plasma with rf source



case 1
case 2
case 3
case 4
ref. 6
ref. 7
ref. 8
ref. 9
ref. 10
ref. 12
ref. 11
ref. 13
ref. 17
ref. 16
ref. 15
ref. 14
$P_o$ (mtorr)
$E_o$ (V/m)
$10^0$
$10^1$
$10^2$
$10^3$